For quick revision of Electrostatics

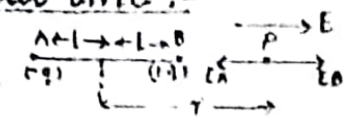
1. ELECTROSTATICS (8 marks)

Electric Field Intensity $E = \frac{1}{4\pi\epsilon_0}\frac{Q}{r^2}$ Vector N/C $E = dV/dr$ | Electric Potential $V = \frac{1}{4\pi\epsilon_0}\frac{q}{r}$ Scalar J/C $V = E/d$, $E = -\frac{dV}{dr}$

ELECTRIC DIPOLE: Equal & opposite charge seprated by small distance. dipole Moment $\vec{P} = 2ql$ Vector unit cm.

1. E and V on Axial Line :-

E at pt P on axial line is



$\vec{E} = \vec{E}_B + (-\vec{E}_A)$

$\vec{E} = \frac{1}{4\pi\epsilon_0}\frac{q}{(r-l)^2} - \frac{1}{4\pi\epsilon_0}\frac{q}{(r+l)^2}$

$\vec{E} = \frac{q}{4\pi\epsilon_0}\left(\frac{1}{(r-l)^2} - \frac{1}{(r+l)^2}\right)$

$\vec{E} = \frac{2\times 2qrl}{4\pi\epsilon_0(r^2-l^2)^2} = \frac{2pr}{4\pi\epsilon_0(r^2-l^2)^2}$

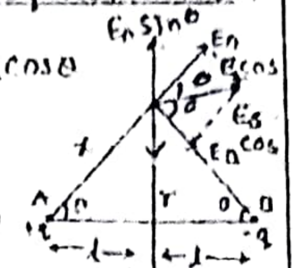
$\boxed{\vec{E} = \frac{2\vec{P}}{4\pi\epsilon_0 r^3}}$ For short dipole $r>>l$ (direction (-) to (+))

$V = V_A + V_B = \frac{1}{4\pi\epsilon_0}\frac{-q}{(r+l)} + \frac{1}{4\pi\epsilon_0}\frac{q}{(r-l)}$

$= \frac{1}{4\pi\epsilon_0}\frac{P}{(r^2-l^2)}$

$\boxed{V = \frac{P}{4\pi\epsilon_0 r^2}}$

2. E and V on Equitorial line:



$\vec{E} = E_A\cos\theta + E_B\cos\theta$

since $E_A = E_B$

$\therefore \vec{E} = 2E_A\cos\theta$

$\vec{E} = 2\times\frac{1}{4\pi\epsilon_0}\frac{q}{x^2}\times\left(\frac{l}{x}\right)$ $\left[\cos\theta = l/x\right]$

$\vec{E} = \frac{2ql}{4\pi\epsilon_0 x^3} = \frac{P}{4\pi\epsilon_0(\sqrt{r^2+l^2})^3}$

$\vec{E} = \frac{P}{4\pi\epsilon_0(r^2+l^2)^{3/2}}$

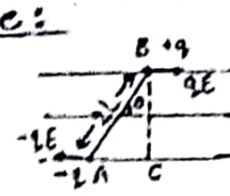
$\boxed{\vec{E} = \frac{1}{4\pi\epsilon_0}\frac{P}{r^3}}$ For short Dipole Direction (+) → (-)

$V = V_A + V_B = \frac{1}{4\pi\epsilon_0}\frac{+q}{x} + \frac{1}{4\pi\epsilon_0}\frac{-q}{x}$

$= 0$

$\boxed{V = 0}$

3. Torque on Dipole:



Net Force

$+qE - qE = 0$

Torque = Force × ⊥ distance

$= qE \times BC$ $[BC = 2l\sin\theta]$

$= qE\, 2l\sin\theta$

$= E(2ql)\sin\theta$

$\boxed{\tau = PE\sin\theta = \vec{P}\times\vec{E}}$

$\tau_{max} = PE$ For $\theta = 90°$

Work Done in Rotating Dipol

$W = \int \tau\, d\theta = (1-\cos\theta)PE$

Energy of Dipole: $U = -PE\cos\theta$

stable equillibrium $\theta = 0$, $U = -PE$

unstable equilibrium

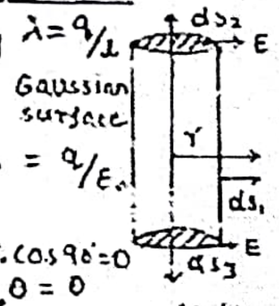
$\theta = 180° \Rightarrow U = PE$.

GAUSS THEOREM:

Total electric flux (total no. of lines of forces) emerges from closed surface is $1/\epsilon_0$ times the charge enclosed. $\oint E.ds = \phi = q/\epsilon_0$

E Due to Long Charged Wire:

Linear charge density $\lambda = q/l$



$\oint E.ds = q/\epsilon_0$

$\int E ds_1 + \int E ds_2 + \int E ds_3 = q/\epsilon_0$

For ds_2 & ds_3 $\theta = 90°$ $\therefore \cos 90° = 0$

For curved surface $ds_1, \theta = 0$

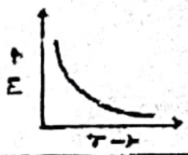
$\int E ds = \int E ds\cos\theta$

$\therefore E\int ds = q/\epsilon_0$

$E(2\pi rl) = q/\epsilon_0$

$E = \frac{q}{2\pi r l\epsilon_0} = \frac{2q/l}{\pi 4\epsilon_0 r}$

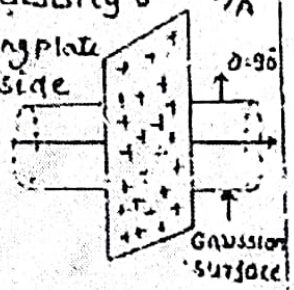
$\boxed{\vec{E} = \frac{1}{4\pi\epsilon_0}\frac{2\lambda}{r}}$



2. E Due to charged Plane sheet:

Surface charge density $\sigma = q/A$

For non conducting plate charge is on both side



$2\int E ds = q/\epsilon_0$

$E\int ds = q/\epsilon_0$

$2EA = q/\epsilon_0$

$E = \frac{q}{2\epsilon_0 A}$ or $\boxed{E = \frac{\sigma}{2\epsilon_0}}$

For conducting sheet $\boxed{E = \frac{\sigma}{\epsilon_0}}$

* $\vec{E}$ is independent of distance From the sheet.

3. E Due charged Hallow sphere:

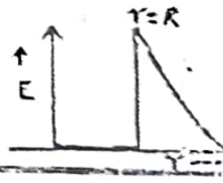
Volume charge density $\rho = q/V$

$\int E ds = q/\epsilon_0$

$E\int ds = q/\epsilon_0$

$E\, 4\pi R^2 = q/\epsilon_0$

$E = \frac{1}{4\pi\epsilon_0}\frac{q}{R^2}$	on surface
$E = \frac{1}{4\pi\epsilon_0}\frac{q}{r^2}$	outside
$\vec{E} = 0$ as $q = 0$	inside



CAPACITOR: $Q = CV$ or $C = Q/V$ Farad, C depends on dimensions.

Capacitance for parallel plate capacitor

Consider ∥ plate capacitor with area of plate A. capacitance C and dist bet plates d

$C = \frac{Q}{V} = \frac{Q}{E\times d}$

$E = \frac{\sigma}{\epsilon_0}$ for charged sheet

$\sigma = \frac{Q}{A}$ for surface charge density

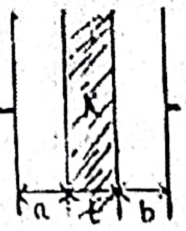
$C = \frac{\sigma A}{\sigma/\epsilon_0 \times d} = \frac{\epsilon_0 A}{d}$

If dielectric with dielectric constant K is filled between the plates.

$\boxed{C = \frac{K\epsilon_0 A}{d}}$

2. With Dielectric slab

Surface charge Density $\sigma = Q/A \therefore Q = \sigma A$



Electric Felld

$\vec{E} = E_{air} + E_{dielectric}$

$= \frac{\sigma}{\epsilon_0} + \frac{\sigma}{K\epsilon_0}$

Potential $V = E\times d$

$\therefore V = \underbrace{\frac{\sigma}{\epsilon_0}(a+b)}_{air} + \underbrace{\frac{\sigma}{K\epsilon_0}t}_{dielectric\ slab}$

$V = \frac{\sigma}{\epsilon_0}(a+b+t/K)$

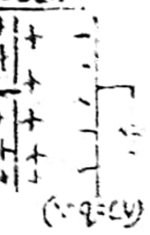
$V = \frac{\sigma}{\epsilon_0}(d-t+t/K)$

Now $C = \frac{Q}{V} = \frac{\sigma A}{\sigma/\epsilon_0(d-t+t/K)}$

$\boxed{C = \frac{A\epsilon_0}{d-t+t/K}}$

3. Energy of Capacitor

Energy = work done in bringing charge at potential V



$dW = V\times dq = \frac{q}{C}.dq$ $(\because q = CV)$

$U = \int_0^Q dW = \frac{1}{C}\int_0^Q q\,dq$

$= \frac{1}{C}\left(\frac{q^2}{2}\right)_0^Q = \frac{1}{2}\frac{Q^2}{C}$

$\boxed{U = \frac{1}{2}\frac{Q^2}{C} = \frac{1}{2}CV^2 = \frac{1}{2}QV}$

Energy Density (Energy per unit volume)

$= \frac{1}{2}CV^2 = \frac{1}{2}\frac{\epsilon_0 A}{d}\frac{(E\times d)^2}{A\times D}$

Volume $= A\times D$ Joule/m^3

$\boxed{U = \frac{1}{2}\epsilon_0 E^2}$